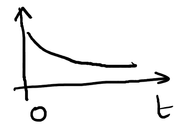
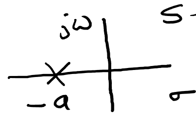
Unit-III - Stability Analysis

Saturday, May 22, 2021 8:34 AM

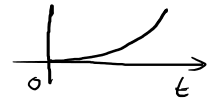
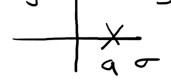
① For bounded input the output is Bounded → stable system.

Eg:- e^{-at} $\quad L\{e^{-at}\} = \frac{1}{s+a}$

stable system

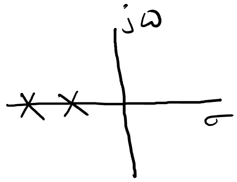
e^{at} $\quad L\{e^{at}\} = \frac{1}{s-a}$

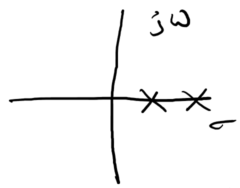
unstable system

② If all poles lies on the Left Half of s-plane — stable system

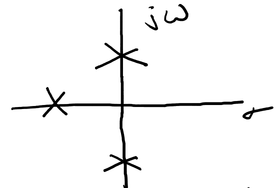
③ If any pole lies on the Right Half of s-plane — unstable system



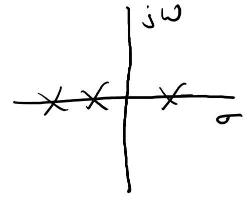
stable



unstable system



Marginally stable
(poles on imaginary axis
No pole on RHS)



unstable

④ Characteristic Equation :- The denominator polynomial of a Transfer function is nothing but characteristic Equation

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$C.E \Rightarrow s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$C.E \Rightarrow a_1 s^n + a_2 s^{n-1} + a_3 s^{n-2} + \dots = 0$

$n \to$ poles.

Routh Hurwitz Criterion — RH criterion (system stability)

Eg:- construct Routh array and comment on stability for the C.E

$$s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$$

Step 1: If the highest power of s is even then first row of routh array contains all coefficients of even powers of 's'. 2nd row contains odd power coeff.

If the highest power of s is odd then the first row of routh array contains all coefficients of odd powers of 's'. 2nd row contains coefficients of even powers of s.

s^4	1	3	5 → Row 1
s^3	2	4	0 → Row 2
s^2	$\frac{2\times3-1\times4}{2} = 1$	$\frac{2\times5-1\times0}{2} = 5$	— Row 3
s^1			— Row 4
s^0			

$$\frac{b_1 \times a_2 - a_1 b_2}{b_1} \qquad \frac{b_1 \times a_3 - a_1 \times b_3}{b_1} \quad \text{— Row 3}$$

Step 2:- $\mid a_1 \quad a_2 \quad a_3$ — Row 1

Step 2 :- compile the Routh Array by performing operation.

$$\begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ & b_1 & b_2 & b_3 \\ & \frac{b_1 \times a_2 - a_1 b_2}{b_1} & \frac{b_1 \times a_3 - a_1 b_3}{b_1} & \end{array}$$

— Row 1
— Row 2
— Row 3

$\frac{b_1 \times a_2 - a_1 b_2}{b_1}$ $\frac{b_1 \times a_3}{b_1}$ — Row 3

Routh Array

$$\begin{array}{c|ccc} s^4 & 1 & 3 & 5 \\ s^3 & 2 & 4 & 0 \\ s^2 & 1 & 5 & \\ s^1 & -6 & 0 & \\ s^0 & 5 & & \end{array}$$

— Row 1
— Row 2
— Row 3
— Row 4
— Row 5

s^2 | $\frac{2\times3 - 1\times4}{2} = 1$ $\frac{2\times5 - 1\times0}{2} = 5$ — Row 3

s^1 | $\frac{1\times4 - 2\times5}{1} = -6$ — Row 4

s^0 | $\frac{-6\times5 - 1\times0}{-6} = 5$ — Row 5

Step 3 :- ① Normal Routh Array (case 1)

If there is no sign changes in the first column of Routh Array then all roots lies on the L.H.S and system is stable.

+1
+2
+1
−6
+5

① sign
② sign change

Total Poles = 4

2 sign changes → 2 Poles R.H.S
→ Remaining 2 Poles L.H.S $= (4-2)$

② If there is sign changes in 1^{st} column of Routh array then system is unstable and the no. of sign changes is Equal to no of poles on then R.H.S.

Result / conclusion :- ① system is unstable.
② Two poles on R.H.S and Two Poles lies on L.H.S

Eg 2:- Comment on stability for the system having C.E

$$s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$$

① Here highest power of s is even. Row 1 → 1 18 5

Row 2 → 8   16

s^4	1	18	5	— Row 1
s^3	8	16	0	— Row 2
s^2	16	5		— Row 3
s^1	13.5	0		— Row 4
s^0	5			— Row 5

s^2 | $\frac{8\times 18 - 1\times 16}{8} = 16$ $\frac{8\times 5 - 1\times 0}{8} = 5$ — Row 3

s^1 | $\frac{16\times 16 - 8\times 5}{16}$ $\frac{256-40}{16}$ ≈ 13.5 — Row 4

s^0 | $\frac{13.5\times 5 - 16\times 0}{13.5} = 5$ — Row 5

Here No sign changes in first column of Routh array.

1. System is stable.
2. All 4 poles lies on L.H.S.

<u>Note</u>:- The stability & location of Poles (No. of sign changes) of system does not change even any row is Multiplied or divided with a Positive number.

$$s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$$

s^4	1	18	5
s^3	1	2	0
s^2	16	5	
s^1	1.7	0	
s^0	5		

s^3 | 8   16 → Row 2

$\frac{8}{8}$ $\frac{16}{8}$

1   2 → Row 2

— — — — — — — —

s^2 | $\frac{1\times 18 - 1\times 2}{1}$ $\frac{1\times 5 - 1\times 0}{1}$ → Row 3

16   5

s^1 | $\frac{16\times 2 - 1\times 5}{16}$ → Row 2

$\frac{27}{16} = 1.7$

s^0 | $\cancel{\frac{1.7\times 5 - 16\times 0}{1.7}}$

No sign changes in First Colum of Routh Array.

1. System is stable
2. All 4 poles Lies on L.H.S

<u>Case 2</u>:- If First element of <u>Row</u> is zero (Replace 0 with ε & $\lim_{\varepsilon \to 0}$)

<u>Eg</u>:- comment on stability and location of poles for system with C.E

$$s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$$

step 1:- Here highest power of 's' is odd. Fill the first row with odd powers of 's' coefficients. Fill 2nd row with coeff of even power.

s^5	1	2	3	— Row 1
s^4	1	2	5	→ Row 2
s^3	ε	-2	0	— Row 3
s^2	$\frac{2\varepsilon+2}{\varepsilon}$	5		— Row 4
s^1	$\frac{-(5\varepsilon^2+4\varepsilon+4)}{2\varepsilon+2}$			— Row 5
s^0	5			→ Row 6

s^3: $\frac{1\times 2-1\times 2}{1}$ $\frac{1\times 3-1\times 5}{1}$

$0 \quad -2$

$\varepsilon \quad -2$ → Row 3

s^2: $\frac{\varepsilon\times 2-1\times(-2)}{\varepsilon}$ $\frac{\varepsilon\times 5-1\times 0}{\varepsilon}$

$\frac{2\varepsilon+2}{\varepsilon} \quad \frac{\cancel{\varepsilon}5}{\cancel{\varepsilon}}$ — Row 4

s^1: $\frac{\left(\frac{2\varepsilon+2}{\varepsilon}\right)\times -2 - 5\varepsilon}{\frac{2\varepsilon+2}{\varepsilon}}$ → Row 5

$= -\frac{(5\varepsilon^2+4\varepsilon+4)}{2\varepsilon+2}$

s^0: $\frac{-\frac{(5\varepsilon^2+4\varepsilon+4)}{2\varepsilon+2}\times 5 - \frac{2\varepsilon+2}{\varepsilon}\times 0}{-\frac{(5\varepsilon^2+4\varepsilon+4)}{2\varepsilon+2}}$ → Row 6

Case of First element of Row zero

Replace zero with 'ε' and complete filling the Routh array. After completion of routh array take $\lim_{\varepsilon\to 0}$ and observe sign changes in First column of Routh array.

$\varepsilon \to 0$

s^5	1	2	3
s^4	1	2	5
s^3	0	-2	
s^2	∞	5	
s^1	-2		
s^0	5		

$+\infty \to -2$

$-2 \to +5$

s^3: $\varepsilon \quad -2$

s^2: $\frac{2\varepsilon+2}{\varepsilon} \quad 5$

$\frac{0+2}{0} = \infty \quad 5$

s^1: $-\frac{(5\varepsilon^2+4\varepsilon+4)}{2\varepsilon+2}$

$-\frac{(5\times 0+4\times 0+4)}{2\times 0+2} = -\frac{4}{2} = -2$

Here Two sign are there in first column of Routh array. so 2 poles lie R.H.S of s-plane. System is unstable

Given system is of order 5. so total 5 poles are there. so poles on L.H.S = 3 (5−2)

Result:- 1) system is unstable

2) 2 Poles on R.H.S & 3 Poles on L.H.S

Case 3:- when all Elements of Row is zero.

① Write the Auxilary polynomial $A(s)$ {Row above the zero Row}

② $\frac{dA(s)}{ds}$. take coefficients in place of zero Row.

③ The roots of Auxilary polynomial are also poles of T.F. This decides whether system is stable or Marginally stable {when No sign changes.}

Eg:- comment on stability and location of Poles for C.E as

$$s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$$

s^6	1	8	20	16	— Row 1
s^5	1	6	8	0	— Row 2
s^4	1	6	8		→ Row 3
s^3	1	3	0		→ Row 4
s^2	$\frac{1\times6-1\times3}{1}=3$	$\frac{1\times8-1\times0}{1}=8$			→ Row 5
s^1	$\frac{3\times3-1\times8}{3}\simeq0.33$				
s^0	$\frac{0.33\times8}{0.33}\simeq8$				

s^6	1	8	20	16
s^5	1	6	8	
s^4	1	6	8	
s^3	1	3	0	
s^2	3	8		
s^1	0.33			
s^0	8			

① No sign changes in first colum. Hence system is

s^5: $\frac{2}{2}$ $\frac{12}{2}$ $\frac{16}{2}$

1 6 8 — Row 2

s^4: $\frac{1\times8-1\times6}{1}$ $\frac{1\times20-1\times8}{1}$ $\frac{1\times16-1\times0}{1}$ → Row 3

2 12 16

$\frac{2}{2}$ $\frac{12}{2}$ $\frac{16}{2}$

1 6 8 — Row 3

s^3: $\frac{1\times6-1\times6}{1}$ $\frac{1\times8-1\times8}{1}$ → Row 4

0 0

Auxilary polynomial

$$A(s) = 1\cdot s^4+6\cdot s^2+8$$

$$A(s) = s^4+6s^2+8$$

$$\frac{dA(s)}{ds} = 4s^3+12s$$

coefficients of $\frac{dA(s)}{ds}$

4 12

$\frac{4}{4}$ $\frac{12}{4}$ {Dividive with 4}

1 3 → Row 4

$A(s) = s^4+6s^2+8$

Roots of $A(s) = ?$ $\quad (s^2) + 6s + 8 = 0$

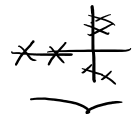
$$s^2 = \frac{-6 \pm \sqrt{36-32}}{2} = \frac{-6 \pm 2}{2}$$

$$s^2 = -2 \text{ or } -4$$

$s^2 = -2 \qquad s^2 = -4$

$s = \pm j\sqrt{2} \qquad s = \pm j2$

1) No sign changes
2) 4 poles on Imaginary Axis



4 poles lies on Imaginary Axis

Result:- 1) system is Marginally stable {4 poles on Imaginary axis}
2) total poles on L.H.S = 2

Procedure for stability Analysis using R-H criterion

① If the highest power of 's' is even then first row of routh array contains all coefficients of even powers of 's'. The second row contains all coefficients of odd powers of 's'.

If the highest power of 's' is odd then first row contains all coefficients of odd powers of 's'. The second row contains all coefficients of even powers of 's'.

Note:- The stability and location of roots of 'CE' do not change even any row is multiplied or divide by a positive number.

Case I:- (Normal Routh Array) {Non-zero elements in 1st column of array}

① complete Routh array.

② If there is no sign change in first column of routh array then all roots lies on Left Half of s-plane and system is stable.

③ If there is sign change in first column of routh array then system is unstable and the number of roots on R.H.S of s-plane is equal to number of sign changes.

Eg:- $+ \; \overset{①}{\frown} \; - \; \overset{②}{\frown} \; + \; + \; + \; \overset{③}{\frown} \; - \; \overset{④}{\frown} \; +$

④ Number of roots on L.H.S = order / Total poles − No. of Poles on RHS

Case II: { First Element of row is zero & Atleast one of remaining elements are non-zero. }

① Replace the zero with smallest positive value 'ε' and continue construction of routh array.

② After completion of routh array substitute $\varepsilon \to 0$ and observe number of sign changes in 1st column.

③ If No sign changes are present → system is stable & All roots on LHS.
If any sign changes are present → system is unstable and number of roots on RHS = No. of sign changes.

Case 3: A row with all Elements as zero

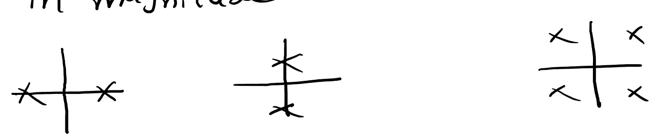
① Obtain the Auxilary polynomial $A(s)$. This is obtained using the row above the zero row.

② Find differentiation of $A(s)$ w.r.t 's' and replace the "zero row" with coefficients of $\frac{dA(s)}{ds}$.

③ Now continue construction of routh array.

④ The roots of Auxilary polynomial are Exists as pairs and with Equal in magnitude and opposite sign.

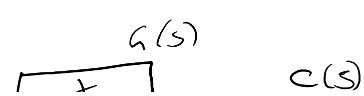
Eg:-



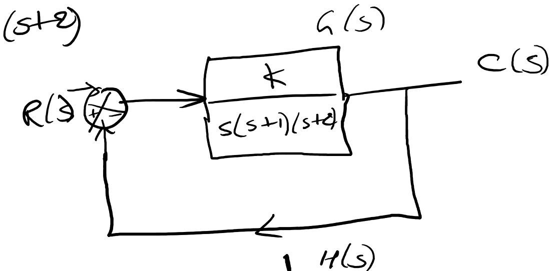
⑤ If there are sign changes in first column. System is unstable. No. of poles on R.H.S = No. of sign changes. The poles lies on imaginary axis is obtained from roots of Auxilary Polynomial $A(s)$. The remaining roots are on L.H.S.

⑥ If there are no sign changes system is Marginally stable and number of roots on L.H.S and imaginary axis are obtained from roots of Auxilary polynomial $A(s)$.

Problem:- Determine range of 'K' for stability of unit feedback system whose open loop T.F Gain $G(s) = \frac{K}{s(s+1)(s+2)}$.



whose open loop T.F Gain $G(s) = \dfrac{\quad}{s(s+1)(s+2)}$.



$$\frac{G(s)}{1+G(s)H(s)}$$

Sol:- $T(s) = \dfrac{C(s)}{R(s)} = \dfrac{\dfrac{k}{s(s+1)(s+2)}}{1 + \dfrac{k}{s(s+1)(s+2)} \cdot 1}$

$$= \frac{\dfrac{k}{\cancel{s(s+1)(s+2)}}}{\dfrac{s(s+1)(s+2)+k}{\cancel{s(s+1)(s+2)}}} = \frac{k}{s(s+1)(s+2)+k}$$

The C.E of system is $s(s+1)(s+2)+k=0$

$$s^3+3s^2+2s+k=0$$

s^3	1	2	→ Row 1
s^2	3	k	→ Row 2
s^1	$\frac{6-k}{3}$		→ Row 3
s^0	K		→ Row 4

$s^1 \quad \dfrac{3\times2-1\times k}{3}$ — Row 3

$\dfrac{6-k}{3}$

$s^0 \quad \dfrac{\cancel{\frac{6-k}{3}}\times k - 3\times0}{\cancel{\frac{6-k}{3}}}$ — Row 4

K

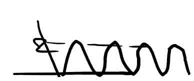
For stable system → the first column of routh array does not have any sign changes.

① $\frac{6-k}{3}>0$ → Row 3
② $k>0$ → Row 4
} conditions for stability of system

① $\frac{6-k}{3}>0 \Rightarrow 6-k>0 \Rightarrow k<6$
② $k>0$
} $0<K<6$

Conclusion:- system is stable for $0<k<6$.

Problem:- For the above system obtain the frequency of oscillations.

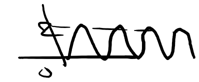
$\xi=1 \rightarrow$ 

Problem:- For the above system obtain the ... oscillations.

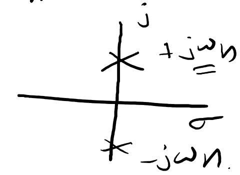
Sol:- ω_n

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \qquad \frac{\omega_n^2}{s^2+\omega_n^2} \qquad s=\pm j\omega_n$$

$\xi=1 \rightarrow$



① To get ω_n; obtain the roots lies on Imaginary axis.



s^3	1	2
s^2	3	K
s^1	$\frac{6-K}{3}$	
s^0	K	

① $K=6$; All zero row is obtain

② $A(s) = 3s^2+K$

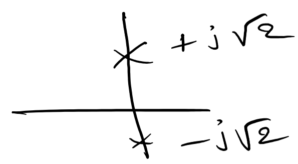
$A(s) = 3s^2+6$ {since $K=6$}

③ $3s^2+6=0$

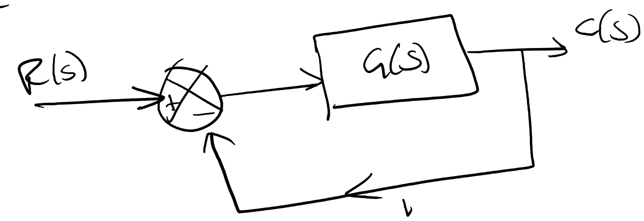
$s^2 = -\frac{6}{3} = -2$ { If $s^2=-2$, $s=\pm j\sqrt{2}$

$\omega_n = \sqrt{2}$

Freq of oscillation is $\sqrt{2}$ rad/sec $K=6$.



Problem:- Given $G(s) = \frac{K}{(s+2)(s+4)(s^2+6s+25)}$ and unit feedback. obtain the K value for stability of system and also find the frequency of oscillation.



Sol:-

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\frac{K}{(s+2)(s+4)(s^2+6s+25)}}{1+\frac{K}{(s+2)(s+4)(s^2+6s+25)}\cdot 1} = \frac{K}{(s+2)(s+4)(s^2+6s+25)+K}$$

C.E of system $(s+2)(s+4)(s^2+6s+25)+K=0$

$$s^4+12s^3+69s^2+198s+(200+K)=0$$

3| 12 198

$$\begin{array}{c|ccc l}
s^4 & 1 & 69 & 200+k & \text{— Row 1} \\
s^3 & 1 & 16.5 & & \text{— Row 2} \\
s^2 & 52.5 & 200+k & & \text{— Row 3} \\
s^1 & \frac{666.25-k}{52.5} & 0 & & \text{— Row 4} \\
s^0 & 200+k & & & \text{— Row 5}
\end{array}$$

$$s^3 \quad \frac{12}{12} \qquad \frac{198}{12}$$

$$1 \qquad 16.5 \rightarrow \text{Row 2}$$

$$s^2 \quad \frac{1\times 69 - 16.5\times 1}{1} \qquad \frac{1\times(200+k) - 1\times 0}{1} \quad \text{— Row 3}$$

$$52.5 \qquad 200+k$$

$$s^1 \quad \frac{52.5\times 16.5 - (200+k)\times 1}{52.5} \quad \text{— Row 4}$$

$$\frac{666.25-k}{52.5}$$

$$s^0 \quad \frac{\frac{666.25-k}{\cancel{52.5}} \times (200+k) - 52.5\times 0}{\cancel{\frac{666.25-k}{52.5}}}$$

① $\frac{666.25-k}{52.5} > 0$

② $200+k > 0$

$666.25 - k > 0$ $\qquad$ $200+k>0$

$k < 666.25$ $\qquad$ $k > -200$

$k > 0$

$\boxed{0 < k < 666.25}$ → For system stability.

② Frequency of oscillations 'ω_n'

① All zero row if $\frac{666.25-k}{52.5} = 0 \Rightarrow k = 666.25$

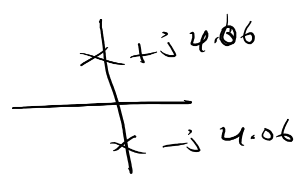
② $A(s) = 52.5s^2 + 200+k$ $\Rightarrow 52.5\, s^2 + 866.25$

$= 52.5s^2 + 200 + 666.25$

$\Rightarrow A(s) = 0$ to get roots

$52.5\, s^2 + 866.25 = 0 \Rightarrow s^2 = \frac{-866.25}{52.5} = -16.5$

$s^2 = -16.5 \Rightarrow s = \pm j\,4.06$

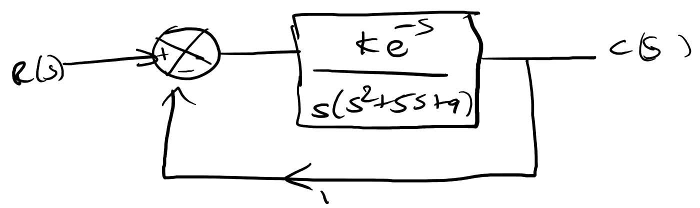


$\boxed{\omega_n = 4.06 \text{ rad/s}}$

... feedback system has open loop function $G(s) = \frac{ke^{-s}}{\ldots}$

Problem:- A unit feed ... $s(s^2+5s+9)$

Determine 'k' value for system to be stable. Also find the frequency of oscillations.



Sol:-

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\cdot H(s)}$$

$$G(s) = \frac{ke^{-s}}{s(s^2+5s+9)}$$

we know $e^{-s} = 1-s+\cancel{\frac{s^2}{2!}-\frac{s^3}{3!}+\cdots}$

$e^{-s} = (1-s)$

$$G(s) = \frac{k(1-s)}{s(s^2+5s+9)}$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{\frac{k(1-s)}{s(s^2+5s+9)}}{1+\frac{k(1-s)}{s(s^2+5s+9)}\cdot 1} = \frac{k(1-s)}{s(s^2+5s+9)+k(1-s)}$$

C.E is $s(s^2+5s+9)+k(1-s)=0$

$$s^3+5s^2+(9-k)s+k=0$$

s^3	1	$9-k$
s^2	5	k
s^1	$\frac{45-6k}{5}$	
s^0	K	

s^1: $\frac{5\times(9-k)-k\times 1}{5}$

$= \frac{45-6k}{5}$

s^0: $\frac{\frac{45-6k}{5}\times k - 5\times 0}{\frac{45-6k}{5}}$

① $\frac{45-6k}{5}>0 \Rightarrow 45-6k>0 \Rightarrow k<7.5$

② $k>0 \Rightarrow k>0$

$\boxed{0<k<7.5}$ - for stable system.

② Frequency of oscillation (ωn).

(1) All zero row $\frac{45-6k}{5} = 0 \Rightarrow k = 7.5$

(2) $A(s) = 5s^2 + k \Rightarrow 5s^2 + 7.5$

(3) Roots of $A(s) \Rightarrow 5s^2 + 7.5 = 0 \Rightarrow 5s^2 = -7.5 \Rightarrow s^2 = \frac{-7.5}{5} = 1.5$

$$\Rightarrow s = \pm j\sqrt{1.5} = \pm j\,1.22$$

$\omega_n = +1.22$ rad/sec

Points to remember

(1) stable system :- Bounded input the output is bounded

(2) Marginally stable system :- For bounded input the output has constant oscillations. This is also called limitedly system.

(3) Asymptotically stable :- Absence of input but output has finite value then it is asymptotically stable.

(4) Absolutely stable system :- system is stable irrespective of the parameter variations.

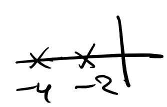
(5) Conditionally stable system :- system is stable for some system parameter variations only. Eg:- $0 < k < 6$ stable

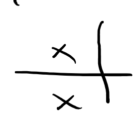
(6) Relative stable system :- It is measure of degree of stability.

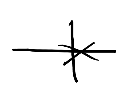
system is BIBO if $\int_{-\infty}^{\infty} h(t) < \infty$.

Some points about stability

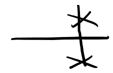
(1) Stable system :-

(1) If Roots are real and negative 

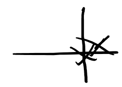
(2) If Roots are complex conjugate in L.H.S of s-plane. 

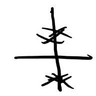
(2) Marginally system :- 

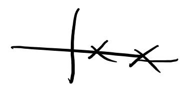
(1) single pole at origin

② Pair of Imaginary poles 

③ Unstable system:-

① Double pole at origin 

② Repeated roots on Imaginary axis 

③ Roots lies on R.H.S 

Note:- How to calculate poles if $s^2 = a+ib$ form.

① Let $s^2 = -\frac{1}{2}+\frac{\sqrt{3}}{2}j$

the two roots obtained as

$-x+iy$ $180^\circ - \tan^{-1}\left(\frac{y}{x}\right)$	$x+iy$ $\tan^{-1}\left(\frac{y}{x}\right)$
$-180^\circ + \tan^{-1}\left(\frac{y}{x}\right)$ $-x-iy$	$-\tan^{-1}\left(\frac{y}{x}\right)$ $x-iy$

$s^2 = -\frac{1}{2}+\frac{\sqrt{3}}{2}j$ (complex form)

$\sqrt{x^2+y^2}\ \angle\theta$ (polar form)

$$s^2 = \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\ \angle 180^\circ - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)$$

$$= 1\ \angle 180^\circ - 60^\circ = 1\ \angle 120^\circ = 1e^{j120^\circ}$$

{ since $e^{j\theta} = \cos\theta + j\sin\theta$ }

$$s^2 = \cos 120^\circ + j\sin 120^\circ$$

$$s = \pm\left[\cos 120^\circ + j\sin 120^\circ\right]^{\frac{1}{2}}$$

$$s = +\left[\cos 120^\circ + j\sin 120^\circ\right]^{\frac{1}{2}} \text{ and } -\left[\cos 120^\circ + j\sin 120^\circ\right]^{\frac{1}{2}}$$

$$= +\left[\cos\frac{120}{2} + j\sin\frac{120}{2}\right] \text{ and } -\left[\cos\frac{120}{2} + j\sin\frac{120}{2}\right]$$

$$= \frac{1}{2}+j\frac{\sqrt{3}}{2} \text{ and } -\left(\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)$$

→ two roots